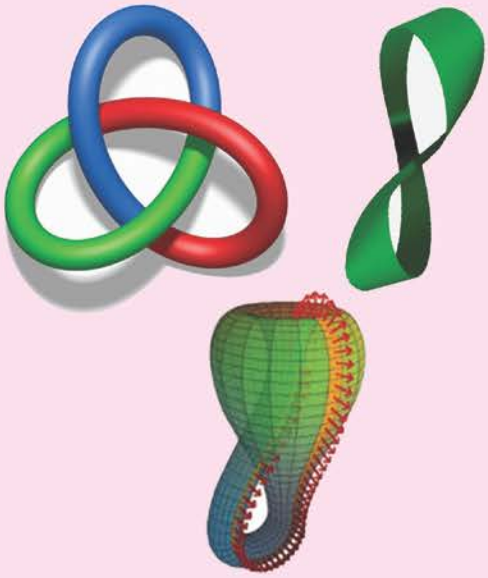
SOME PROBLEMS IN TOPOLOGY

MINOR RESEARCH PROJECT

submitted to

UNIVERSITY GRANTS COMMISSION



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POLYCRISP L - FUZZY TOPOLOGY

Minor Research Project Report

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September 2015

Declaration

I, Vinitha.T hereby declare that this project entitled ‘Some Problems in Topology’ contains no material which had been accepted for any other Degree, Diploma or similar titles in any institution or University and to the best of my knowledge and belief, it contains no material previously published by any person except where due references are made in the text of the project.

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PREFACE

A detailed and comprehensive study of fuzzy topology is reported in this dissertation. Chapter1 is the introductory chapter in which the basic preliminary ideas useful throughout this project are cited. In chapter 2 and 3 a detailed study of fuzzy topology was done; the concepts like fuzzy set, fuzzy topology, neighbourhood of a fuzzy point in a fuzzy topological space, fuzzy continuous functions, fuzzy separation axioms etc were discussed in detail. A detailed review of the present work is given in chapter 4 and Chapter 5.

Significance of '0' and '1' in the definition of a crisp set in usual set theory, leads to the introduction of a new concept called poly crisp L-fuzzy set that can admit any two values of a lattice . Chapter 4 mainly deals with the definition of poly crisp L-fuzzy set and detailed study of its set theoretic properties. Application of poly crispness in a topological structure is also discussed. Poly crisp L-fuzzy topology is introduced in this chapter. Concepts like p-open sets, p-interior and p-closure operator are studied in detail in chapter 4.Application of this new notion in subspace topology and continuous functions between two topological spaces is studied in chapter 5.The notion of polycrisp quotient topology, p-continuity ,direct sum between two p-fts is also viewed in detail.

Acknowledgement

First of all I am grateful to God for all his blessings in life which made me what I am today.

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I am very much grateful to Dr.AnitaNair, Principal , Al-Ameen College, Edathala for encouraging me to apply for and also completing the project successfully. I am also grateful to my colleagues especially the faculty members in the Dept. of Mathematics for their sincere co-operation and help through out my project work.

VINITHA.T

Introduction

Fuzzy set was introduced by Zadeh in 1965. In mathematics, a set A can be equivalently represented by its characteristic function that is a mapping $\chi_A(x)$ from the universe X containing A to the 2-valued set{0,1},but in fuzzy case the "belonging to" relation $\chi_A(x)$ between 'x' and 'A' is no longer '0' or otherwise '1',it has a degree of membership, such as 0.7.Therefore the range has to be extended from {0,1} to [0,1] or more generally a lattice L, because all the membership degrees in mathematical view ,form an ordered structure, a lattice. Thus a fuzzy set on a universe X is simply just a mapping from X to a lattice L. Fuzzy topology is just a kind of topology developed on fuzzy sets. Fuzzy topological spaces were first introduced by Chang in 1968.Ordinary sets are actually characteristic function with membership values '0' or '1',where '0' denotes NO and '1' denotes YES. But is there any significance for '0' or '1';should it be replaced by some 'α' and 'β'. This question motivated us to concentrate on a class of fuzzy sets that have certain properties in common. Thus we try to introduce and study polycrisp L-fuzzy sets and polycrisp L- fuzzy topological spaces and some of its basic notions. The results in this project are foundations for a systematic treatment of polycrisp L- fuzzy topological spaces.

Main findings are:

- **Polycrisp L-fuzzy topological space**

Let X be any ordinary set and let L be any chained lattice and A be any collection of L- fuzzy sets on X,then A is said to form a polycrisp L-fuzzy topology on X if A satisfies the following 4 conditions:

(1)$\underline{1}$, $\underline{0}$$\in$ A

(2)$\forall A \subset$ A,$\vee A \in$ A

(3)$\forall U$, $V\in$ A,$U \wedge V \in$ A

(4)$\forall A \in$ A, A must be a polycrisp subset.

Then (X,A) is said to form a polycrisp L-fuzzy topological space.Every element in A is called a polycrisp open fuzzy subset in (X,A).

- **Neighbourhood of a point in a p-fts**

Let (X,T) be a p-fts.Then for every x in X we can define polycrispneighbourhood of $x \in X$ .A polycrisp set $N \in L^X$ is a neighbourhood of x in X if and only if $\exists$ $G \in T$ such that $G \leq N$ and N(y)=G(x)when y=x

and N(y)=G(z) for some $z \in X$ when $y \neq x$.

- **Neighborhood of a fuzzy set in a p-fts**

Let (X,T) be a p-fts and let $A \in L^X$.A polycrisp fuzzy set $H \in L^X$ is a p-nbd of $A \in L^X$ if and only if $\exists$ $G \in T$ such that $A \leq G \leq H$ and $A(x) = G(y)$ for some $y \in X$.

- **Subspace Topology**

  Let (X,T) be a p-fts;$Y \subset X, Y \neq \phi$.Then$T|_y = \{A|_y : A \in T\}$ will form a polycrisp fuzzy topology. This polycrisp L-fuzzy topology is called relative polycrisp fuzzy topology or subspace polycrisp topology of Y and $(Y,T|_Y)$ is called polycrisp L- fuzzy subspace.

- **Mapping Between Polycrisp Fuzzy Sets**

Let X and Y be 2 ordinary sets and 'f' be a mapping between X and Y and A,B be 2 polycrisp L-fuzzy sets in X and Y respectively ;define polycrisp L fuzzy mapping from set of all polycrisp subsets of X to the set of all polycrisp subsets of Y by

$f(A)(y) = \vee\{A(x)/x \in X, f(x) = y\}$ when $f^{-1}(y) \neq \phi$ and $f(A)(y) = \wedge\{A(x)/x \in X\}$ when $f^{-1}(y) = \phi$ and define the polycrisp L- fuzzy inverse mapping as $f{-}1(B)(x) = B(f(x)); x \in X$.

- **Continuity between 2 p-fts**

Let (X,T), (Y,S) 2 p-fts,also let f be a polycrisp fuzzy mapping between (X,T) and(Y,S),then f is polycrisp continuous if and only if whenever $B \in S$ then $f^{-1}(B) \in T$.

- **Polycrisp quotient topology**

Let (X,T) be a p-fts and let $f : X \to Y$ where Y an ordinary set and f is onto. Define the polycrisp quotient topology $T|_f$ of T with respect to f by $T|_f = V \in L^Y / f^{-1}(V) \in T$.

CONCLUSION

Fuzzy topological spaces were first introduced by Chang in 1968. Ordinary sets are actually characteristic function with membership values ‘0’ or ‘1’,where ‘0’ denotes NO and ‘1’ denotes YES. In this project I introduced and study polycrisp L-fuzzy sets and polycrisp L- fuzzy topological spaces and some of its basic notions. There are still remaining many topological properties like separation axioms, compactification etc that has to be analysed in the context of poly-crispness.